Advanced Numerical Analysis

General Course Information

Prerequisites: Numerical Analysis (I) and (II)

Note: Most important material of the course will be presented in class, so attendance is imperative.

Textbooks:

- K. E. Atkinson and W. Han, Theoretical Numerical Analysis: *A Functional Analysis Framework* (2nd Ed.), Springer (2005).
- J. Stoer and R. Bulrisch, Introduction to Numerical Analysis (2nd Ed.), Springer (1993).
- E. S¨uli and D. F. Mayers, An Introduction to Numerical Analysis, Cambridge University Press (2003).
- G. Dahlquist and A. Bj¨orck, Numerical Methods in Scientific Computing, SIAM (2007).
- P. Linz, Theoretical Numerical Analysis, John Wiley (1979).
- A. Quarteroni, R. Sacco and F. Saleri, Numerical Mathematics, Springer (2000).
- K. E. Atkinson, An Introduction to Numerical Analysis (2nd Ed.), John Wiley (1989).

A Tentative Course Outline:

1. **Basic Concepts in Error Estimation**

   Sources of Errors - Fixed and Floating Point Representation - IEEE Floating Point Standard- Multiple Precision Arithmetic - Accuracy and Rounding Errors - Basic Rounding Error – Avoiding Overflow and Cancellation - Error Propagation - Propagation of Errors and Condition Numbers - Error Analysis and Stability of Algorithms - Automatic Control of Accuracy and Verified Computing - Running Error Analysis - Interval Arithmetic - Interval Matrix Computations.

2. **Some Topics in Interpolation and Approximation**

   The Interpolation Problem - Bases for Polynomial Interpolation - Conditioning of Polynomial Interpolation - Interpolation Formulas and Algorithms - Newton's Interpolation Formula - Inverse Interpolation - Barycentric Lagrange Interpolation - Fast Algorithms for Vandermonde Systems - The Runge Phenomenon - Hermite Interpolation – Rational Interpolation - Multivariate Interpolation - Piecewise Polynomial Interpolation – Spline Functions - The B-Spline Basis - Least Squares Splines Approximation – Mathematical Properties of Orthogonal Polynomials - Expansions in Orthogonal Polynomials - Approximation in the Maximum Norm - Fourier Methods - Discrete Fourier Analysis – Convergence Acceleration of Fourier Series - The Fast Fourier Transform – Discrete Convolution by FFT- FFT's of Real Data - Fast Trigonometric Transforms.

3. Numerical Integration

Interpolatory Quadrature Rules - Some Classical Formulas - Higher Order Newton–Cotes' Formulas - Clenshaw–Curtis Rules - Integration by Extrapolation – Euler Maclaurin's Formula - Romberg's Method - Adaptive Quadrature - Quadrature Rules with Free Nodes – Gauss Christoffel Quadrature Rules - Peano kernel Theorem - Weighted Gaussian Quadratures and its Error Analysis – Radau and Lobatto Quadrature - Product Rules - Singular Integrals.

4. Solving Scalar Nonlinear Equations

Some Basic Concepts and Methods - Limiting Accuracy and Termination Criteria - Fixed Point Iteration - Convergence Order and Efficiency - Methods Based on Interpolation - The Secant Method - Higher Order Interpolating Methods - Methods Using Derivatives - Newton's Method - Newton's Method for Complex Roots - An Interval Newton Method - Higher Order Methods - Finding a Minimum of a Function - Unimodal Functions and Golden Section Search - Minimization by Interpolation - Algebraic Equations – Deflation and Simultaneous Determination of Roots - A Modified Newton Method – Sturm Sequences - Finding Greatest Common Divisors.

Assessment:

Homework and class presentation	20%
Midterm	30%
Final Exam	50%